

**Sebastián Martín Ruiz**

**Applications of Smarandache Function, and  
Prime and Coprime Functions**

$$C_k(n_1, n_2, \dots, n_k) = \begin{cases} 0 & \text{if } n_1, n_2, \dots, n_k \text{ are coprime numbers} \\ 1 & \text{otherwise} \end{cases}$$

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## Chapter 1: Smarandache function applied to perfect numbers

The Smarandache function is defined as follows:

$S(n)$  = the smallest positive integer such that  $S(n)!$  is divisible by  $n$ . [1]

In this article we are going to see that the value this function takes when  $n$  is a perfect number of the form  $n = 2^{k-1} \cdot (2^k - 1)$ ,  $p = 2^k - 1$  being a prime number.

Lemma 1: Let  $n = 2^i \cdot p$  when  $p$  is an odd prime number and  $i$  an integer such that:

$$0 \leq i \leq E\left(\frac{p}{2}\right) + E\left(\frac{p}{2^2}\right) + E\left(\frac{p}{2^3}\right) + \dots + E\left(\frac{p}{2^{E(\log_2 p)}}\right) = e_2(p!)$$

where  $e_2(p!)$  is the exponent of 2 in the prime number decomposition of  $p!$ .

$E(x)$  is the greatest integer less than or equal to  $x$ .

One has that  $S(n) = p$ .

Demonstration:

Given that  $GCD(2^i, p) = 1$  (GCD= greatest common divisor) one has that

$S(n) = \max\{S(2^i), S(p)\} \geq S(p) = p$ . Therefore  $S(n) \geq p$ .

If we prove that  $p!$  is divisible by  $n$  then one would have the equality.

$$p! = p_1^{e_{p_1}(p!)} \cdot p_2^{e_{p_2}(p!)} \cdot \dots \cdot p_s^{e_{p_s}(p!)}$$

where  $p_i$  is the  $i$ -th prime of the prime number decomposition of  $p!$ . It is clear that

$p_1 = 2$ ,  $p_s = p$ ,  $e_{p_s}(p!) = 1$  for which:

$$p! = 2^{e_2(p!)} \cdot p_2^{e_{p_2}(p!)} \cdot \dots \cdot p_{s-1}^{e_{p_{s-1}}(p!)} \cdot p$$

From where one can deduce that:

$$\frac{p!}{n} = 2^{e_2(p!)-i} \cdot p_2^{e_{p_2}(p!)} \dots p_{s-1}^{e_{p_{s-1}}(p!)}$$

is a positive integer since  $e_2(p!) - i \geq 0$ .

Therefore one has that  $S(n) = p$

Proposition1: If  $n$  is a perfect number of the form  $n = 2^{k-1} \cdot (2^k - 1)$  with  $k$  is a positive integer,  $2^k - 1 = p$  prime, one has that  $S(n) = p$ .

Demonstration:

For the Lemma it is sufficient to prove that  $k - 1 \leq e_2(p!)$ .

If we can prove that:

$$k - 1 \leq 2^{k-1} - \frac{1}{2} \quad (1)$$

we will have proof of the proposition since:

$$k - 1 \leq 2^{k-1} - \frac{1}{2} = \frac{2^k - 1}{2} = \frac{p}{2}$$

As  $k - 1$  is an integer one has that  $k - 1 \leq E\left(\frac{p}{2}\right) \leq e_2(p!)$

Proving (1) is the same as proving  $k \leq 2^{k-1} + \frac{1}{2}$  at the same time, since  $k$  is integer, is equivalent to proving  $k \leq 2^{k-1}$  (2).

In order to prove (2) we may consider the function:  $f(x) = 2^{x-1} - x$   $x$  real number.

This function may be derived and its derivate is  $f'(x) = 2^{x-1} \ln 2 - 1$ .

$f$  will be increasing when  $2^{x-1} \ln 2 - 1 > 0$  resolving  $x$ :

$$x > 1 - \frac{\ln(\ln 2)}{\ln 2} \cong 1.5287$$

In particular  $f$  will be increasing  $\forall x \geq 2$ .

Therefore  $\forall x \geq 2$   $f(x) \geq f(2) = 0$  that is to say  $2^{x-1} - x \geq 0$   $\forall x \geq 2$ .

Therefore:  $2^{k-1} \geq k \forall k \geq 2$  integer.

And thus is proved the proposition.

EXAMPLES:

$6 = 2 \cdot 3$	$S(6)=3$
$28 = 2^2 \cdot 7$	$S(28)=7$
$496 = 2^4 \cdot 31$	$S(496)=31$
$8128 = 2^6 \cdot 127$	$S(8128)=127$

References:

[1] C. Dumitrescu and R. Müller: To Enjoy is a Permanent Component of Mathematics. SMARANDACHE NOTIONS JOURNAL Vol. 9 No 1-2, (1998) pp 21-26

## Chapter 2: A result obtained using the Smarandache Function

Smarandache Function is defined as followed:

$S(m)$ =The smallest positive integer so that  $S(m)!$  is divisible by  $m$ . [1]

Let's see the value which such function takes for  $m = p^{n^2}$  with  $n$  integer,  $n \geq 2$  and  $p$  prime number. To do so a Lemma required.

Lemma 1  $\forall m, n \in \mathbb{N} \quad m, n \geq 2$

$$m^n = E\left[\frac{m^{n+1} - m^n + m}{m}\right] + E\left[\frac{m^{n+1} - m^n + m}{m^2}\right] + \dots + E\left[\frac{m^{n+1} - m^n + m}{m^{E[\log_m(m^{n+1} - m^n + m)]}}\right]$$

where  $E(x)$  gives the greatest integer less than or equal to  $x$ .

Proof:

Let's see in the first place the value taken by  $E[\log_m(m^{n+1} - m^n + m)]$ .

If  $n \geq 2$ :  $m^{n+1} - m^n + m < m^{n+1}$  and therefore

$$\log_m(m^{n+1} - m^n + m) < \log_m m^{n+1} = n + 1.$$

And if  $m \geq 2$ :

$$mm^n \geq 2m^n \Rightarrow m^{n+1} \geq 2m^n \Rightarrow m^{n+1} + m \geq 2m^n \Rightarrow m^{n+1} - m^n + m \geq m^n \\ \Rightarrow \log_m(m^{n+1} - m^n + m) \geq \log_m m^n = n \Rightarrow E[\log_m(m^{n+1} - m^n + m)] \geq n$$

As a result:  $n \leq E[\log_m(m^{n+1} - m^n + m)] < n + 1$  therefore:

$$E[\log_m(m^{n+1} - m^n + m)] = n \quad \text{if } n, m \geq 2$$

Now let's see the value which it takes for  $1 \leq k \leq n$ :  $E\left[\frac{m^{n+1} - m^n + m}{m^k}\right]$

$$E\left[\frac{m^{n+1} - m^n + m}{m^k}\right] = E\left[m^{n+1-k} - m^{n-k} + \frac{1}{m^{k-1}}\right]$$



$$\text{If } k=1: E\left[\frac{m^{n+1} - m^n + m}{m^k}\right] = m^n - m^{n-1} + 1$$

$$\text{If } 1 < k \leq n: E\left[\frac{m^{n+1} - m^n + m}{m^k}\right] = m^{n+1-k} - m^{n-k}$$

Let's see what is the value of the sum:

$$k=1 \quad m^n \quad -m^{n-1} \quad \dots \quad \dots \quad \dots \quad \dots \quad +1$$

$$k=2 \quad \quad m^{n-1} \quad -m^{n-2}$$

$$k=3 \quad \quad \quad m^{n-2} \quad -m^{n-3}$$

$$\cdot \quad \quad \quad \cdot$$

$$\cdot \quad \quad \quad \cdot$$

$$\cdot \quad \quad \quad \cdot$$

$$k=n-1 \quad \quad \quad \quad \quad \quad m^2 \quad -m$$

$$k=n \quad \quad \quad \quad \quad \quad m \quad -1$$

Therefore:

$$\sum_{k=1}^n E\left[\frac{m^{n+1} - m^n + m}{m^k}\right] = m^n \quad m, n \geq 2$$

Proposition:  $\forall p$  prime number  $\forall n \geq 2$ :

$$S(p^{p^n}) = p^{n+1} - p^n + p$$

Proof:

Having  $e_p(k)$  = exponent of the prime number  $p$  in the prime decomposition of  $k$ .

We get:

$$e_p(k) = E\left(\frac{k}{p}\right) + E\left(\frac{k}{p^2}\right) + E\left(\frac{k}{p^3}\right) + \dots + E\left(\frac{k}{p^{E(\log_p k)}}\right)$$

And using the lemma we have

$$e_p \left[ (p^{n+1} - p^n + p)! \right] = E \left[ \frac{p^{n+1} - p^n + p}{p} \right] + E \left[ \frac{p^{n+1} - p^n + p}{p^2} \right] + \dots + E \left[ \frac{p^{n+1} - p^n + p}{p^{E \left[ \log_p (p^{n+1} - p^n + p) \right]}} \right] = p^n$$

Therefore:

$$\frac{(p^{n+1} - p^n + p)!}{p^{p^n}} \in \mathbb{N} \quad \text{and} \quad \frac{(p^{n+1} - p^n + p - 1)!}{p^{p^n}} \notin \mathbb{N}$$

And :

$$S(p^{p^n}) = p^{n+1} - p^n + p$$

References:

[1] C. Dumitrescu and R. Müller: To Enjoy is a Permanent Component of Mathematics. SMARANDACHE NOTIONS JOURNAL VOL 9:, No. 1-2 (1998) pp 21-26.

## Chapter 3: A Congruence with the Smarandache function

Smarandache's function is defined thus:

$S(n)$  = is the smallest integer such that  $S(n)!$  is divisible by  $n$ . [1]

In this article we are going to look at the value that has  $S(2^k - 1) \pmod{k}$

For all integer,  $2 \leq k \leq 97$ .

$k$	$S(2^k - 1)$	$S(2^k - 1) \pmod{k}$
2	3	1
3	7	1
4	5	1
5	31	1
6	7	1
7	127	1
8	17	1
9	73	1
10	31	1
11	89	1
12	13	1
13	8191	1
14	127	1
15	151	1
16	257	1
17	131071	1
18	73	1
19	524287	1
20	41	1
21	337	1
22	683	1
23	178481	1
24	241	1
25	1801	1
26	8191	1
27	262657	1
28	127	15
29	2089	1
30	331	1

k	$S(2^k-1)$	$S(2^k-1) \pmod k$
31	2147483647	1
32	65537	1
33	599479	1
34	131071	1
35	122921	1
36	109	1
37	616318177	1
38	524287	1
39	121369	1
40	61681	1
41	164511353	1
42	5419	1
43	2099863	1
44	2113	1
45	23311	1
46	2796203	1
47	13264529	1
48	673	1
49	4432676798593	1
50	4051	1
51	131071	1
52	8191	27
53	20394401	1
54	262657	1
55	201961	1
56	15790321	1
57	1212847	1
58	3033169	1
59	3203431780337	1
60	1321	1
61	2305843009213693951	1
62	2147483647	1
63	649657	1
64	6700417	1
65	145295143558111	1
66	599479	1
67	761838257287	1
68	131071	35

k	$S(2^k-1)$	$S(2^k-1) \pmod k$
69	10052678938039	1
70	122921	1
71	212885833	1
72	38737	1
73	9361973132609	1
74	616318177	1
75	10567201	1
76	525313	1
77	581283643249112959	1
78	22366891	1
79	1113491139767	1
80	4278255361	1
81	97685839	1
82	8831418697	1
83	57912614113275649087721	1
84	14449	1
85	9520972806333758431	1
86	2932031007403	1
87	9857737155463	1
88	2931542417	1
89	618970019642690137449562111	1
90	18837001	1
91	23140471537	1
92	2796203	47
93	658812288653553079	1
94	165768537521	1
95	30327152671	1
96	22253377	1
97	13842607235828485645766393	1

One can see from the table that there are only 4 exceptions for  $2 \leq k \leq 97$

We can see in detail the 4 exceptions in a table:

$k=28=2^2 \cdot 7$	$S(2^{28}-1) \equiv 15 \pmod{28}$
$k=52=2^2 \cdot 13$	$S(2^{52}-1) \equiv 27 \pmod{52}$
$k=68=2^2 \cdot 17$	$S(2^{68}-1) \equiv 35 \pmod{68}$
$k=92=2^2 \cdot 23$	$S(2^{92}-1) \equiv 47 \pmod{92}$

One can observe in these 4 cases that  $k=2^2p$  with  $p$  is a prime and more over  $S(2^k - 1) \equiv \frac{k}{2} + 1 \pmod{k}$

UNSOLVED QUESTION:

One can obtain a general formula that gives us, in function of  $k$  the value  $S(2^k - 1) \pmod{k}$  for all positive integer values of  $k$ ?

Reference:

[1] Smarandache Notions Journal, Vol. 9, No. 1-2, (1998), pp. 21-26.

## Chapter 4: A functional recurrence to obtain the prime numbers using the Smarandache prime function.

Theorem: We are considering the function:

For  $n$  integer:

$$F(n) = n + 1 + \sum_{m=n+1}^{2n} \prod_{i=n+1}^m \left[ - \frac{\sum_{j=1}^i \left( \left\lfloor \frac{i}{j} \right\rfloor - \left\lfloor \frac{i-1}{j} \right\rfloor \right) - 2}{i} \right]$$

one has:  $p_{k+1} = F(p_k)$  for all  $k \geq 1$  where  $\{p_k\}_{k \geq 1}$  are the prime numbers and  $\lfloor x \rfloor$  is the greatest integer less than or equal to  $x$ .

Observe that the knowledge of  $p_{k+1}$  only depends on knowledge of  $p_k$  and the knowledge of the fore primes is unnecessary.

Proof:

Suppose that we have found a function  $P(i)$  with the following property:

$$P(i) = \begin{cases} 1 & \text{if } i \text{ is composite} \\ 0 & \text{if } i \text{ is prime} \end{cases}$$

This function is called Smarandache prime function.(Ref.)

Consider the following product:

$$\prod_{i=p_k+1}^m P(i)$$

If  $p_k < m < p_{k+1}$   $\prod_{i=p_k+1}^m P(i) = 1$  since  $i: p_k + 1 \leq i \leq m$  are all composites.

If  $m \geq p_{k+1}$   $\prod_{i=p_k+1}^m P(i) = 0$  since  $P(p_{k+1}) = 0$

Here is the sum:

$$\begin{aligned} \sum_{m=p_k+1}^{2p_k} \prod_{i=p_k+1}^m P(i) &= \sum_{m=p_k+1}^{p_{k+1}-1} \prod_{i=p_k+1}^m P(i) + \sum_{m=p_{k+1}}^{2p_k} \prod_{i=p_k+1}^m P(i) = \sum_{m=p_k+1}^{p_{k+1}-1} 1 = \\ &= p_{k+1} - 1 - (p_k + 1) + 1 = p_{k+1} - p_k - 1 \end{aligned}$$

The second sum is zero since all products have the factor  $P(p_{k+1}) = 0$ .

Therefore we have the following recurrence relation:

$$p_{k+1} = p_k + 1 + \sum_{m=p_k+1}^{2p_k} \prod_{i=p_k+1}^m P(i)$$

Let's now see we can find  $P(i)$  with the asked property.

Consider:

$$\left\lfloor \frac{i}{j} \right\rfloor - \left\lfloor \frac{i-1}{j} \right\rfloor = \begin{cases} 1 & \text{si } j \mid i \\ 0 & \text{si } j \text{ not } \mid i \end{cases} \quad j = 1, 2, \dots, i \quad i \geq 1$$

We deduce of this relation:

$$d(i) = \sum_{j=1}^i \left\lfloor \frac{i}{j} \right\rfloor - \left\lfloor \frac{i-1}{j} \right\rfloor$$

where  $d(i)$  is the number of divisors of  $i$ .



If  $i$  is prime  $d(i) = 2$  therefore:

$$-\left\lfloor -\frac{d(i)-2}{i} \right\rfloor = 0$$

If  $i$  is composite  $d(i) > 2$  therefore:

$$0 < \frac{d(i)-2}{i} < 1 \Rightarrow -\left\lfloor -\frac{d(i)-2}{i} \right\rfloor = 1$$

Therefore we have obtained the Smarandache Prime Function  $P(i)$  which is:

$$P(i) = -\left\lfloor -\frac{\sum_{j=1}^i \left( \left\lfloor \frac{i}{j} \right\rfloor - \left\lfloor \frac{i-1}{j} \right\rfloor \right) - 2}{i} \right\rfloor \quad i \geq 2 \text{ integer}$$

With this, the theorem is already proved .

References:

[1] E. Burton, "Smarandache Prime and Coprime functions".

[www.gallup.unm.edu/~Smarandache/primfct.txt](http://www.gallup.unm.edu/~Smarandache/primfct.txt)

[2]F. Smarandache, "Collected Papers", Vol II 200, p.p. 137, Kishinev University Press, Kishinev, 1997.

## Chapter 5: The general term of the prime number sequence and the Smarandache prime function.

Let us consider the function  $d(i)$  = number of divisors of the positive integer number  $i$ . We have found the following expression for this function:

$$d(i) = \sum_{k=1}^i E\left(\frac{i}{k}\right) - E\left(\frac{i-1}{k}\right)$$

“ $E(x) = \text{Floor}[x]$ ”

We proved this expression in the article “A functional recurrence to obtain the prime numbers using the Smarandache Prime Function”.

We deduce that the following function:

$$G(i) = -E\left[-\frac{d(i)-2}{i}\right]$$

This function is called the Smarandache Prime Function (Reference)

It takes the next values:

$$G(i) = \begin{cases} 0 & \text{if } i \text{ is prime} \\ 1 & \text{if } i \text{ is composite} \end{cases}$$

Let us consider now  $\pi(n)$  = number of prime numbers smaller or equal than  $n$ .

It is simple to prove that:

$$\pi(n) = \sum_{i=2}^n (1 - G(i))$$

Let us have too:

$$\begin{aligned} \text{If } 1 \leq k \leq p_n - 1 &\Rightarrow E\left(\frac{\pi(k)}{n}\right) = 0 \\ \text{If } C_n \geq k \geq p_n &\Rightarrow E\left(\frac{\pi(k)}{n}\right) = 1 \end{aligned}$$

We will see what conditions have to carry  $C_n$ .

Therefore we have the following expression for  $p_n$  n-th prime number:

$$p_n = 1 + \sum_{k=1}^{C_n} \left(1 - E\left(\frac{\pi(k)}{n}\right)\right)$$

If we obtain  $C_n$  that only depends on  $n$ , this expression will be the general term of the prime numbers sequence, since  $\pi$  is in function with  $G$  and  $G$  does with  $d(i)$  that is expressed in function with  $i$  too. Therefore the expression only depends on  $n$ .

Let us consider  $C_n = 2(E(n \log n) + 1)$

Since  $p_n \approx n \log n$  from a certain  $n_0$  it will be true that

$$(1) \quad p_n \leq 2(E(n \log n) + 1)$$

If  $n_0$  it is not too big, we can prove that the inequality is true for smaller or equal values than  $n_0$ .

It is necessary to that:

$$E\left[\frac{\pi(2(E(n \log n) + 1))}{n}\right] = 1$$

If we check the inequality:

$$(2) \quad \pi(2(E(n \log n) + 1)) < 2n$$

We will obtain that:

$$\frac{\pi(C_n)}{n} < 2 \Rightarrow E\left[\frac{\pi(C_n)}{n}\right] \leq 1 \quad ; \quad C_n \geq p_n \Rightarrow E\left[\frac{\pi(C_n)}{n}\right] = 1$$

We can experimentally check this last inequality saying that it checks for a lot of values and the difference tends to increase, wich makes to think that it is true for all n .

Therefore if we prove that the (1) and (2) inequalities are true for all n which seems to be very probable; we will have that the general term of the prime numbers sequence is:

$$p_n = 1 + \sum_{k=1}^{2(E(n \log n)+1)} \left[ 1 - E \left[ \frac{\sum_{j=2}^k \left[ 1 + E \left[ - \frac{\sum_{s=1}^j (E(j/s) - E((j-1)/s)) - 2}{j} \right]}{n} \right]}{n} \right] \right]$$

Reference:

[1] E. Burton, "Smarandache Prime and Coprime Functions"

[Http://www.gallup.unm.edu/~Smarandache/primfnct.txt](http://www.gallup.unm.edu/~Smarandache/primfnct.txt)

[2] F. Smarandache, "Collected Papers", Vol. II, 200 p.,p.137, Kishinev University Press.

## Chapter 6: Expressions of the Smarandache Coprime Function

Smarandache Coprime function is defined this way:

$$C_k(n_1, n_2, \dots, n_k) = \begin{cases} 0 & \text{if } n_1, n_2, \dots, n_k \text{ are coprime numbers} \\ 1 & \text{otherwise} \end{cases}$$

We see two expressions of the Smarandache Coprime Function for  $k=2$ .

EXPRESSION 1:

$$C_2(n_1, n_2) = \left\lfloor -\frac{n_1 n_2 - \text{lcm}(n_1, n_2)}{n_1 n_2} \right\rfloor$$

$\lfloor x \rfloor$  = the biggest integer number smaller or equal than  $x$ .

If  $n_1, n_2$  are coprime numbers:

$$\text{lcm}(n_1, n_2) = n_1 n_2 \quad \text{therefore:} \quad C_2(n_1, n_2) = \left\lfloor \frac{0}{n_1 n_2} \right\rfloor = 0$$

If  $n_1, n_2$  aren't coprime numbers:

$$\text{lcm}(n_1, n_2) < n_1 n_2 \Rightarrow 0 < \frac{n_1 n_2 - \text{lcm}(n_1, n_2)}{n_1 n_2} < 1 \Rightarrow C_2(n_1, n_2) = 1$$

EXPRESSION 2:

$$C_2(n_1, n_2) = 1 + \left\lfloor -\frac{\prod_{\substack{d|n_1 \\ d>1}} \prod_{\substack{d'|n_2 \\ d'>1}} |d - d'|}{\prod_{d|n_1} \prod_{d'|n_2} (d + d')} \right\rfloor$$

If  $n_1, n_2$  are coprime numbers then  $d \neq d' \quad \forall d, d' \neq 1$

$$\Rightarrow 0 < \frac{\prod_{d|n_1} \prod_{d'|n_2} |d - d'|}{\prod_{d>1} \prod_{d'>1} (d + d')} < 1 \Rightarrow C_2(n_1, n_2) = 0$$

If  $n_1, n_2$  aren't coprime numbers  $\exists d = d' \quad d > 1, d' > 1 \Rightarrow C_2(n_1, n_2) = 1$

EXPRESSION 3:

Smarandache Coprime Function for  $k \geq 2$ :

$$C_k(n_1, n_2, \dots, n_k) = \left\lfloor \frac{1}{GCD(n_1, n_2, \dots, n_k)} - 1 \right\rfloor$$

If  $n_1, n_2, \dots, n_k$  are coprime numbers:

$$GCD(n_1, n_2, \dots, n_k) = 1 \Rightarrow C_k(n_1, n_2, \dots, n_k) = 0$$

If  $n_1, n_2, \dots, n_k$  aren't coprime numbers:  $GCD(n_1, n_2, \dots, n_k) > 1$

$$0 < \frac{1}{GCD} < 1 \Rightarrow \left\lfloor \frac{1}{GCD} - 1 \right\rfloor = 1 = C_k(n_1, n_2, \dots, n_k)$$

References:

1. E. Burton, "Smarandache Prime and Coprime Function"
2. F. Smarandache, "Collected Papers", Vol II 22 p.p. 137, Kishinev University Press.

## Chapter 7: New Prime Numbers

I have found some new prime numbers using the PROTH program of Yves Gallot.

This program is based on the following theorem:

### **Proth Theorem (1878):**

Let  $N = k \cdot 2^n + 1$  where  $k < 2^n$ . If there is an integer number  $a$  so that  $a^{\frac{N-1}{2}} \equiv -1 \pmod{N}$  therefore  $N$  is prime.

The Proth program is a test for primality of greater numbers defined as  $k \cdot b^n + 1$  or  $k \cdot b^n - 1$ . The program is made to look for numbers of less than 5.000000 digits and it is optimized for numbers of more than 1000 digits..

Using this Program, I have found the following prime numbers:

$3239 \cdot 2^{12345} + 1$	with 3720 digits	$a = 3, a = 7$
$7551 \cdot 2^{12345} + 1$	with 3721 digits	$a = 5, a = 7$
$7595 \cdot 2^{12345} + 1$	with 3721 digits	$a = 3, a = 11$
$9363 \cdot 2^{12321} + 1$	with 3713 digits	$a = 5, a = 7$

Since the exponents of the first three numbers are Smarandache number  $Sm(5)=12345$  we can call this type of prime numbers, prime numbers of Smarandache .

Helped by the MATHEMATICA program, I have also found new prime numbers which are a variant of prime numbers of Fermat. They are the following:

$$2^{2^n} \cdot 3^{2^n} - 2^{2^n} - 3^{2^n} \text{ for } n=1, 4, 5, 7 .$$

It is important to mention that for  $n=7$  the number which is obtained has 100 digits.

Chris Nash has verified the values  $n=8$  to  $n=20$ , this last one being a number of 815.951 digits, obtaining that they are all composite. All of them have a tiny factor except  $n=13$ .

***References:***

1. Micha Fleuren, “Smarandache Factors and Reverse Factors”, Smarandache Notions Journal, Vol. 10, [www.gallup.unm.edu/~smarandache/](http://www.gallup.unm.edu/~smarandache/)
2. Chris Caldwell, The Prime Pages, [www.utm.edu/research/primes](http://www.utm.edu/research/primes)



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